MAT 201 - DOĞRUSAL CEBİR UYGULAMA NOTLARI ①

SORU:
$$\left.\begin{aligned} x-2y+z&=2\\ 2x+y-z&=1\\ -3x+y-2z&=-5 \end{aligned}\right\}$$
Lineer denklem sistemini eliminasyon (yok etme) metoduyla çözünüz.

ÇÖZÜM: İlk olarak 1. denklemden hareketle 2. ve 3. denklemdeki x'li terimi yok edelim. Bunun için 1. denklemin (-2) katını 2. denkleme, (3) katını 3. denkleme ilave edelim.

$$\begin{aligned} x-2y+z&=2\\ 5y-3z&=-3\\ -5y+z&=1 \end{aligned}$$

Şimdi 2. denklemden hareketle: 3. denklemdeki y'li terimi yok edelim. Bunun için 2. denklemin (1) katını 3. denkleme ilave edelim.

$$\begin{aligned} x-2y+z&=2\\ 5y-3z&=-3\\ -2z&=-2 \end{aligned}$$

Sonuç olarak; 3. denklemden $z=1$ olarak bulunur. Bu değeri 2. denklemde yerine yazarsak $y=0$ ve bu değerleri 1. denklemde yerine yazarsak $x=1$ bulunur. $(x,y,z)=(1,0,1)$

SORU:
$$A=\begin{bmatrix} 2 & 1 & -2\\ 3 & 4 & 5 \end{bmatrix},\quad B=\begin{bmatrix} 3 & -1\\ -2 & 3\\ 4 & 5 \end{bmatrix},\quad C=\begin{bmatrix} 3 & -1 & 3\\ 4 & 1 & 5\\ 2 & 1 & 3 \end{bmatrix}$$

$$D=\begin{bmatrix} -2 & 1\\ 3 & 4 \end{bmatrix},\quad E=\begin{bmatrix} 2 & -3 & 4\\ 1 & -2 & 3\\ 4 & 1 & 0 \end{bmatrix},\quad F=\begin{bmatrix} -1 & 2\\ 0 & 4\\ 3 & 5 \end{bmatrix}$$

matrisleri veriliyor. Buna göre; a) AB b) BA c) F^TE d) $CB+D$ e) $AB+D^2$ hesaplayınız

ÇÖZÜM

ⓐ $$AB=\begin{bmatrix} 2 & 1 & -2\\ 3 & 4 & 5 \end{bmatrix}\begin{bmatrix} 3 & -1\\ -2 & 3\\ 4 & 5 \end{bmatrix}=\begin{bmatrix} 2.3+1.(-2)+(-2).4 & 2.(-1)+1.3+(-2).5\\ 3.3+4.(-2)+5.4 & 3(-1)+4.3+5.5 \end{bmatrix}=\begin{bmatrix} -4 & -9\\ 21 & 34 \end{bmatrix}$$

ⓑ $$BA=\begin{bmatrix} 3 & -1\\ -2 & 3\\ 4 & 5 \end{bmatrix}\begin{bmatrix} 2 & 1 & -2\\ 3 & 4 & 5 \end{bmatrix}=\begin{bmatrix} 3 & -1 & -11\\ 5 & 10 & 19\\ 23 & 24 & 17 \end{bmatrix}$$

ⓒ $$F^TE=\begin{bmatrix} -1 & 0 & 3\\ 2 & 4 & 5 \end{bmatrix}\begin{bmatrix} 2 & -3 & 4\\ 1 & -2 & 3\\ 4 & 1 & 0 \end{bmatrix}=\begin{bmatrix} (-1).2+0.1+3.4 & (-1)(-3)+0.(-2)+3.1 & (-1).4+0.3+3.0\\ 2.2+4.1+5.4 & 2.(-3)+4.(-2)+5.1 & 2.4+4.3+5.0 \end{bmatrix}=\begin{bmatrix} 10 & 6 & -\\ 28 & -9 & 2 \end{bmatrix}$$

ⓓ $CB+D$ hesaplanamaz! Çünkü $(CB)_{3\times2}$ ve $(D)_{2\times2}$ nin toplanabilmesi için aynı sayıda satır ve sütuna sahip olmaları gerekirdi.

ⓔ $$AB+D^2=\begin{bmatrix} -4 & -9\\ 21 & 34 \end{bmatrix}+\begin{bmatrix} -2 & 1\\ 3 & 4 \end{bmatrix}\begin{bmatrix} -2 & 1\\ 3 & 4 \end{bmatrix}=\begin{bmatrix} -4 & -9\\ 21 & 34 \end{bmatrix}+\begin{bmatrix} 7 & 2\\ 6 & 19 \end{bmatrix}=\begin{bmatrix} -4+7 & -9+2\\ 21+6 & 34+19 \end{bmatrix}=\begin{bmatrix} 3 & -7\\ 27 & 53 \end{bmatrix}$$

(ÖDEV)

SORU: $A=\begin{bmatrix}2&1&1\\0&3&2\\3&4&5\end{bmatrix}$ ve $B=\begin{bmatrix}6&2&0\\1&3&2\\4&3&2\end{bmatrix}$ matrisleri verilsin. $(AB)^T=B^TA^T$ olduğunu gösteriniz.

ÇÖZÜM: $A^T=\begin{bmatrix}2&0&3\\1&3&4\\1&2&5\end{bmatrix}$, $B^T=\begin{bmatrix}6&1&4\\2&3&3\\0&2&2\end{bmatrix} \Rightarrow B^TA^T=\begin{bmatrix}6&1&4\\2&3&3\\0&2&2\end{bmatrix}\begin{bmatrix}2&0&3\\1&3&4\\1&2&5\end{bmatrix}=\begin{bmatrix}17&11&42\\10&15&33\\4&10&18\end{bmatrix}$

$AB=\begin{bmatrix}2&1&1\\0&3&2\\3&4&5\end{bmatrix}\begin{bmatrix}6&2&0\\1&3&2\\4&3&2\end{bmatrix}=\begin{bmatrix}17&10&4\\11&15&10\\42&33&18\end{bmatrix} \Rightarrow (AB)^T=\begin{bmatrix}17&11&42\\10&15&33\\4&10&18\end{bmatrix}$

SORU $A=\begin{bmatrix}3&2\\-1&4\end{bmatrix}$ kare matrisi ters çevrilebilir (tekil olmayan) olsun. $b=\begin{bmatrix}4\\-6\end{bmatrix}$ için:

$Ax=b$ lineer sistemin çözümünü bulunuz.

ÇÖZÜM: (A ters çevrilebilir $\overset{\text{kare}}{\text{matris}}$ $\Rightarrow$ $Ax=b$ lineer sisteminin $x=A^{-1}b$ o.s. tek çözümü vardır.)

$A=\begin{bmatrix}3&2\\-1&4\end{bmatrix}$, $A^{-1}=\begin{bmatrix}a&b\\c&d\end{bmatrix} \Rightarrow AA^{-1}=I_2$ olmalıdır. (A, ters çevrilebilir olduğundan)

$AA^{-1}=\begin{bmatrix}3&2\\-1&4\end{bmatrix}\begin{bmatrix}a&b\\c&d\end{bmatrix}=\begin{bmatrix}1&0\\0&1\end{bmatrix} \Rightarrow \begin{bmatrix}3a+2c&3b+2d\\-a+4c&-b+4d\end{bmatrix}=\begin{bmatrix}1&0\\0&1\end{bmatrix}$

$\Rightarrow \begin{cases}3a+2c=1\\-a+4c=0\end{cases}$ $\Rightarrow c=1/14$, $\Rightarrow a=2/7$ $\Rightarrow A^{-1}=\begin{bmatrix}2/7&-1/7\\1/14&3/14\end{bmatrix}$

$\begin{cases}3b+2d=0\\-b+4d=1\end{cases}$ $\Rightarrow d=3/14$, $\Rightarrow b=-1/7$

$x=A^{-1}b=\begin{bmatrix}2/7&-1/7\\1/14&3/14\end{bmatrix}\begin{bmatrix}4\\-6\end{bmatrix}=\begin{bmatrix}2\\-1\end{bmatrix}$ olarak bulunur.

SORU: a) $A=\begin{bmatrix}-1&2&-5\\2&-1&6\\1&1&3\end{bmatrix}$, elemanter satır işlemlerini kullanarak A yı eşelon forma indirge

b) $A=\begin{bmatrix}1&-2&3\\0&1&2\\-2&9&4\end{bmatrix}$, 〃 A yı indirgenmiş eşelon forma indirge

ÇÖZÜM:

(a) $\begin{bmatrix}-1&2&-5\\2&-1&6\\1&1&3\end{bmatrix} \xrightarrow{-r_1\to r_1} \begin{bmatrix}1&-2&5\\2&-1&6\\1&1&3\end{bmatrix} \xrightarrow[-r_1+r_3\to r_3]{-2r_1+r_2\to r_2} \begin{bmatrix}1&-2&5\\0&3&-4\\0&3&-2\end{bmatrix} \xrightarrow{\frac{1}{3}r_2\to r_2} \begin{bmatrix}1&-2&5\\0&1&-\frac{4}{3}\\0&3&-2\end{bmatrix}$

$\xrightarrow{-3r_2+r_3\to r_3} \begin{bmatrix}1&-2&5\\0&1&-\frac{4}{3}\\0&0&2\end{bmatrix} \xrightarrow{\frac{1}{2}r_3\to r_3} \begin{bmatrix}1&-2&5\\0&1&-\frac{4}{3}\\0&0&1\end{bmatrix}$

(b) $\begin{bmatrix}1&-2&3\\0&1&2\\-2&9&4\end{bmatrix} \xrightarrow{2r_1+r_3\to r_3} \begin{bmatrix}1&-2&3\\0&1&2\\0&5&10\end{bmatrix} \xrightarrow{-5r_2+r_3\to r_3} \begin{bmatrix}1&-2&3\\0&1&2\\0&0&0\end{bmatrix} \xrightarrow{+2r_2+r_1\to r_1} \begin{bmatrix}1&0&7\\0&1&2\\0&0&0\end{bmatrix}$

SORU: $\left.\begin{array}{l} x-2y+z=2 \\ 2x+y-z=1 \\ -3x+y-2z=-5 \end{array}\right\}$ lineer denklem sistemini eşelon forma indirgeyip çözünüz. (Gauss-Eliminasyon)

ÇÖZÜM.

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 2 \\ 2 & 1 & -1 & 1 \\ -3 & 1 & -2 & -5 \end{array}\right] \xrightarrow[3r_1+r_3\to r_3]{-2r_1+r_2\to r_2} \left[\begin{array}{ccc|c} 1 & -2 & 1 & 2 \\ 0 & 5 & -3 & -3 \\ 0 & -5 & 1 & 1 \end{array}\right] \xrightarrow{\frac{1}{5}r_2\to r_2} \left[\begin{array}{ccc|c} 1 & -2 & 1 & 2 \\ 0 & 1 & -\frac{3}{5} & -\frac{3}{5} \\ 0 & -5 & 1 & 1 \end{array}\right]$$

$$\xrightarrow{5r_2+r_3\to r_3} \left[\begin{array}{ccc|c} 1 & -2 & 1 & 2 \\ 0 & 1 & -\frac{3}{5} & -\frac{3}{5} \\ 0 & 0 & -2 & -2 \end{array}\right] \xrightarrow{-\frac{1}{2}r_3\to r_3} \left[\begin{array}{ccc|c} 1 & -2 & 1 & 2 \\ 0 & 1 & -\frac{3}{5} & -\frac{3}{5} \\ 0 & 0 & 1 & 1 \end{array}\right] \Rightarrow \left.\begin{array}{r} x-2y+z=2 \\ y-\frac{3}{5}z=-\frac{3}{5} \\ z=1 \end{array}\right\} \Rightarrow (x,y,z) = (1,0,1)$$

SORU $\left.\begin{array}{l} x+2y+3z=-1 \\ x-2y-z=-1 \\ 3x+y+z=3 \end{array}\right\}$ a) l.d.s. Gauss-Eliminasyon metoduyla çözünüz. b) l.d.s. Gauss-Jordan metoduyla çözünüz.

ÇÖZÜM

a) $$\left[\begin{array}{ccc|c} 1 & 2 & 3 & -1 \\ 1 & -2 & -1 & -1 \\ 3 & 1 & 1 & 3 \end{array}\right] \xrightarrow[-3r_1+r_3\to r_3]{-r_1+r_2\to r_2} \left[\begin{array}{ccc|c} 1 & 2 & 3 & -1 \\ 0 & -4 & -4 & 0 \\ 0 & -5 & -8 & 6 \end{array}\right] \xrightarrow{-r_3+r_2\to r_2} \left[\begin{array}{ccc|c} 1 & 2 & 3 & -1 \\ 0 & 1 & 4 & -6 \\ 0 & -5 & -8 & 6 \end{array}\right]$$

$$\xrightarrow{5r_2+r_3\to r_3} \left[\begin{array}{ccc|c} 1 & 2 & 3 & -1 \\ 0 & 1 & 4 & -6 \\ 0 & 0 & 12 & -24 \end{array}\right] \xrightarrow{\frac{1}{12}r_3\to r_3} \underbrace{\left[\begin{array}{ccc|c} 1 & 2 & 3 & -1 \\ 0 & 1 & 4 & -6 \\ 0 & 0 & 1 & -2 \end{array}\right]}_{\text{EŞELON MATRİS}} \Rightarrow \left.\begin{array}{r} x+2y+3z=-1 \\ y+4z=-6 \\ z=-2 \end{array}\right\} \Rightarrow (x,y,z) = (1,2,-2)$$

b) ... $$\underbrace{\left[\begin{array}{ccc|c} 1 & 2 & 3 & -1 \\ 0 & 1 & 4 & -6 \\ 0 & 0 & 1 & -2 \end{array}\right]}_{\text{EŞELON MATRİS}} \xrightarrow[-4r_3+r_2\to r_2]{-3r_3+r_1\to r_1} \left[\begin{array}{ccc|c} 1 & 2 & 0 & 5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -2 \end{array}\right] \xrightarrow{-2r_2+r_1\to r_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -2 \end{array}\right] \begin{array}{l} \to x \\ \to y \\ \to z \end{array}$$

SORU $\left.\begin{array}{l} x+y=4 \\ x+(a^2-15)y=a \end{array}\right\}$ l.d.s. a nın durumuna göre inceleyiniz. (Ne zaman çözüm yok, ne zaman çözüm var, ne zaman sonsuz çözüm...)

ÇÖZÜM

$$\left[\begin{array}{cc|c} 1 & 1 & 4 \\ 1 & a^2-15 & a \end{array}\right] \xrightarrow{-r_1+r_2\to r_2} \left[\begin{array}{cc|c} 1 & 1 & 4 \\ 0 & a^2-16 & a-4 \end{array}\right] \Rightarrow \begin{cases} x+y=4 \\ (a^2-16)y=a-4 \end{cases}$$

- $a^2-16=0 \Rightarrow a=\mp 4$
  - $a=4 \Rightarrow (0.y=0)$ her y için sağlanır $\Rightarrow$ Sonsuz çözüm var.
  - $a=-4 \Rightarrow (0.y=-8$ durumu hiçbir y için sağlanmaz$) \Rightarrow$ çözüm yok!
- $a^2-16\neq 0 \Rightarrow a\neq\mp 4 \Rightarrow$ sistemin tek çözümü vardır.

SORU:
$$\left.\begin{aligned} x - 2y - z &= -2 \\ 2x + y + 3z &= 1 \\ -3x + y - 2z &= 1 \end{aligned}\right\}$$
ldd. Gauss-Eliminasyon metodu ile çözünüz.

ÇÖZÜM:
$$\left[\begin{array}{ccc:c} 1 & -2 & -1 & -2 \\ 2 & 1 & 3 & 1 \\ -3 & 1 & -2 & 1 \end{array}\right] \xrightarrow[3r_1+r_3 \to r_3]{-2r_1+r_2 \to r_2} \left[\begin{array}{ccc:c} 1 & -2 & -1 & -2 \\ 0 & 5 & 5 & 5 \\ 0 & -5 & -5 & -5 \end{array}\right]$$

$$\xrightarrow{\frac{1}{5}r_2 \to r_2} \left[\begin{array}{ccc:c} 1 & -2 & -1 & -2 \\ 0 & 1 & 1 & 1 \\ 0 & -5 & -5 & -5 \end{array}\right] \xrightarrow{5r_2+r_3 \to r_3} \left[\begin{array}{ccc:c} 1 & -2 & -1 & -2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \left.\begin{aligned} x-2y-z &= -2 \\ y+z &= 1 \\ 0.z &= 0 \end{aligned}\right\} \to \text{her } z \text{ içi[n] sağlanır}$$

$\Rightarrow$ $z=t$ dersek; $y = 1-t$, $x = -t$ bulunur. Yani; t-parametresine bağlı sonsuz çözüm va[r]

SORU:
$$\left.\begin{aligned} x + y &= 4 \\ x + (a^2-15)y &= a \end{aligned}\right\}$$
ldd ni a nın durumuna göre inceleyiniz.

ÇÖZÜM
$$\left[\begin{array}{cc:c} 1 & 1 & 4 \\ 1 & a^2-15 & a \end{array}\right] \xrightarrow{-r_1+r_2 \to r_2} \left[\begin{array}{cc:c} 1 & 1 & 4 \\ 0 & a^2-16 & a-4 \end{array}\right] \Rightarrow$$

- $a^2-16=0 \Rightarrow a = \mp 4$
  - $a=4 \Rightarrow$ Son[suz] çöz[üm]
  - $a=-4 \Rightarrow$ çöz[üm] y[ok]
- $a^2-16 \neq 0 \Rightarrow a \neq \mp 4 \Rightarrow$ tek çöz[üm]

(ÖDEV) SORU.
$$\left.\begin{aligned} x + y + z &= 3 \\ x + 2y + z &= 3 \\ x + y + (a^2-8)z &= a \end{aligned}\right\}$$
ldd ni a nın durumuna göre inceleyiniz.

ÇÖZÜM
$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 3 \\ 1 & 2 & 1 & 3 \\ 1 & 1 & a^2-8 & a \end{array}\right] \xrightarrow[-r_1+r_3 \to r_3]{-r_1+r_2 \to r_2} \left[\begin{array}{ccc:c} 1 & 1 & 1 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & a^2-9 & a-3 \end{array}\right] \Rightarrow$$

- $a^2-9=0 \Rightarrow a = \mp 3$
  - $a=3 \Rightarrow$ So[nsuz] çö[züm]
  - $a=-3 \Rightarrow$ ç[özüm] y[ok]
- $a^2-9 \neq 0 \Rightarrow a \neq \mp 3 \Rightarrow$ tek çöz[üm]

ÖDEV:
$$\left.\begin{aligned} x + y + 2z &= a \\ x \quad\;\; + z &= b \\ 2x + y + 3z &= c \end{aligned}\right\}$$
ldd nin çözümünün olması için a, b, c arasındaki bağıntı ne olmalıdır?

ÇÖZÜM:
$$\left[\begin{array}{ccc:c} 1 & 1 & 2 & a \\ 1 & 0 & 1 & b \\ 2 & 1 & 3 & c \end{array}\right] \xrightarrow[-2r_1+r_3 \to r_3]{-r_1+r_2 \to r_2} \left[\begin{array}{ccc:c} 1 & 1 & 2 & a \\ 0 & -1 & -1 & b-a \\ 0 & -1 & -1 & -2a+c \end{array}\right] \xrightarrow{-r_2 \to r_2} \left[\begin{array}{ccc:c} 1 & 1 & 2 & a \\ 0 & 1 & 1 & a-b \\ 0 & -1 & -1 & -2a+c \end{array}\right]$$

$$\xrightarrow{r_2+r_3 \to r_3} \left[\begin{array}{ccc:c} 1 & 1 & 2 & a \\ 0 & 1 & 1 & a-b \\ 0 & 0 & 0 & c-a-b \end{array}\right] \Rightarrow$$

- $c-a-b=0 \Rightarrow$ sonsuz çözüm ✓
- $c-a-b \neq 0 \Rightarrow$ çözüm yok

O halde çözümün olması için; $c-a-b=0$ olmalıdır.

SORU: $A=\begin{bmatrix}1&3&3\\1&4&3\\1&3&4\end{bmatrix}$ ters çevrilebilir matris olsun.

a) A matrisinin tersini bulunuz. (elemanter satır işlemleriyle)

b) A matrisinin tersini; elemanter matrislerin çarpımı şeklinde yazınız.

c) A matrisini; elemanter matrislerin çarpımı şeklinde yazınız.

ÇÖZÜM. A kare matrisinin ters çevrilebilir olması için $\Longleftrightarrow$ A matrisi birim matrise dönüştürülebilir

(a) $[A:I] \xrightarrow{\cdots} [I:A^{-1}]$

$$[A:I]=\left[\begin{array}{ccc|ccc}1&3&3&1&0&0\\1&4&3&0&1&0\\1&3&4&0&0&1\end{array}\right]\xrightarrow{-r_1+r_2\to r_2}\left[\begin{array}{ccc|ccc}1&3&3&1&0&0\\0&1&0&-1&1&0\\1&3&4&0&0&1\end{array}\right]\xrightarrow{-r_1+r_3\to r_3}\left[\begin{array}{ccc|ccc}1&3&3&1&0&0\\0&1&0&-1&1&0\\0&0&1&-1&0&1\end{array}\right]$$

$$\xrightarrow{-3r_3+r_1\to r_1}\left[\begin{array}{ccc|ccc}1&3&0&4&0&-3\\0&1&0&-1&1&0\\0&0&1&-1&0&1\end{array}\right]\xrightarrow{-3r_2+r_1\to r_1}\left[\begin{array}{ccc|ccc}1&0&0&7&-3&-3\\0&1&0&-1&1&0\\0&0&1&-1&0&1\end{array}\right]=[I:A^{-1}]$$

(b) A ters çevrilebilir matris olduğunda, A elemanter matrislerin çarpımı şeklinde yazılabilir.

$\underbrace{E_k\dots E_2E_1}_{A^{-1}}A=I \Longrightarrow A^{-1}=E_k\cdots E_2E_1$ ve $A=E_1^{-1}E_2^{-1}\cdots E_k^{-1}$ olarak yazılabilir.

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\xrightarrow{-r_1+r_2\to r_2}\begin{bmatrix}1&0&0\\-1&1&0\\0&0&1\end{bmatrix}=E_1$$

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\xrightarrow{-r_1+r_3\to r_3}\begin{bmatrix}1&0&0\\0&1&0\\-1&0&1\end{bmatrix}=E_2$$

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\xrightarrow{-3r_3+r_1\to r_1}\begin{bmatrix}1&0&-3\\0&1&0\\0&0&1\end{bmatrix}=E_3$$

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\xrightarrow{-3r_2+r_1\to r_1}\begin{bmatrix}1&-3&0\\0&1&0\\0&0&1\end{bmatrix}=E_4$$

$$A^{-1}=E_4E_3E_2E_1=\begin{bmatrix}7&-3&-3\\-1&1&0\\-1&0&1\end{bmatrix}$$

(c) E_{ij}; $r_i\longleftrightarrow r_j$ elemanter satır işlemine karşılık gelen elemanter matris $\Longrightarrow (E_{ij})^{-1}=E_{ij}$

$E_i(\alpha)$ ($\alpha\neq 0$); $\alpha r_i\longrightarrow r_i$ 〃 $\Longrightarrow (E_i(\alpha))^{-1}=E_i(\frac{1}{\alpha})$

$E_{ij}(\alpha)$ ($\alpha\neq 0$); $\alpha r_i+r_j\to r_j$ 〃 $\Longrightarrow (E_{ij}(\alpha))^{-1}=E_{ij}(-\alpha)$

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\xrightarrow{r_1+r_2\to r_2}\begin{bmatrix}1&0&0\\1&1&0\\0&0&1\end{bmatrix}=E_1^{-1}$$

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\xrightarrow{r_1+r_3\to r_3}\begin{bmatrix}1&0&0\\0&1&0\\1&0&1\end{bmatrix}=E_2^{-1}$$

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\xrightarrow{3r_3+r_1\to r_1}\begin{bmatrix}1&0&3\\0&1&0\\0&0&1\end{bmatrix}=E_3^{-1}$$

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\xrightarrow{3r_2+r_1\to r_1}\begin{bmatrix}1&3&0\\0&1&0\\0&0&1\end{bmatrix}=E_4^{-1}$$

$$A=E_1^{-1}E_2^{-1}E_3^{-1}E_4^{-1}$$

SORU $A=\begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & -2 \\ -5 & 4 & -7 \end{bmatrix}$ matrisinin tersi var mıdır?

Çözüm:

$$[A:I]=\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ -1 & 2 & -2 & 0 & 1 & 0 \\ -5 & 4 & -7 & 0 & 0 & 1 \end{array}\right] \xrightarrow[5r_1+r_3\to r_3]{r_1+r_2\to r_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 2 & -1 & 1 & 1 & 0 \\ 0 & 4 & -2 & 5 & 0 & 1 \end{array}\right] \xrightarrow{-2r_2+r_3\to r_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 2 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 3 & -2 & 1 \end{array}\right]$$

Sıfır satırına sahip olduğu için A'nın tersi yoktur!

SORU: $A=\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 2 & a \end{bmatrix}$ olsun. Hangi a değerleri için A^{-1} vardır? $A^{-1}=?$

Çözüm:

$$[A:I]=\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 2 & a & 0 & 0 & 1 \end{array}\right] \xrightarrow[-r_1+r_3\to r_3]{-r_1+r_2\to r_2} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & -1 & 1 & 0 \\ 0 & 1 & a & -1 & 0 & 1 \end{array}\right] \xrightarrow{-r_2\to r_2} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 1 & a & -1 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{-r_2+r_3\to r_3} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & a & -2 & 1 & 1 \end{array}\right] \xrightarrow[\frac{1}{a}r_3\to r_3,\ a\neq 0]{-r_2+r_1\to r_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & \frac{-2}{a} & \frac{1}{a} & \frac{1}{a} \end{array}\right] = [I:A^{-1}]$$

$a\neq 0$ için A^{-1} vardır!

SORU:

$$\left.\begin{aligned} x+z &= 1 \\ 2x+y &= 2 \\ -y+z &= 3 \end{aligned}\right\}$$ lineer denklem sistemini ters matris yöntemi ile çözünüz.

Çözüm: Verilen lineer denklem sisteminin katsayılar matrisi; $A=\begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$ olup,

bu matrisin tersini bulalım.

$$[A:I]=\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{-2r_1+r_2\to r_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & -2 & -2 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{r_2+r_3\to r_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & -2 & -2 & 1 & 0 \\ 0 & 0 & -1 & -2 & 1 & 1 \end{array}\right]$$

$$\xrightarrow{-r_3\to r_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & -2 & -2 & 1 & 0 \\ 0 & 0 & 1 & 2 & -1 & -1 \end{array}\right] \xrightarrow[-r_3+r_1\to r_1]{2r_3+r_2\to r_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & 1 \\ 0 & 1 & 0 & 2 & -1 & -2 \\ 0 & 0 & 1 & 2 & -1 & -1 \end{array}\right] = [I:A^{-1}]$$

$$A^{-1}=\begin{bmatrix} -1 & 1 & 1 \\ 2 & -1 & -2 \\ 2 & -1 & -1 \end{bmatrix}$$ bulunur.

O halde $AX=B$ ve A ters çevrilebilir matris $\Rightarrow X=A^{-1}B$ çözümdür.

$$X=\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} -1 & 1 & 1 \\ 2 & -1 & -2 \\ 2 & -1 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}=\begin{bmatrix} 4 \\ -6 \\ -3 \end{bmatrix}\begin{array}{l} \to x \\ \to y \\ \to z \end{array}$$